AFRL-RD-PS-
TR-2014-0045

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TR-2014-0045

Novel Detection of Optical Orbital Angular Momentum

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16 Nov 2014

Final Report

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REPORT DOCUMENTATION PAGE	*Form Approved* *OMB No. 0704-0188*

1. REPORT DATE *(DD-MM-YYYY)*	2. REPORT TYPE	3. DATES COVERED *(From - To)*
22-09-2014	Final Report	11/16/12-8/14/14

4. TITLE AND SUBTITLE
Novel Detection of Optical Orbital Angular Momentum

5a. CONTRACT NUMBER FA9451-13-1-0261

5b. GRANT NUMBER GR0004113

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)
David Voelz

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER D04R

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Klipsch School of ECE
New Mexico State University
MSC 3-O, PO Box 30001
Las Cruces, NM 88003

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Air Force Research Laboratory
3550 Aberdeen Ave SE
Kirtland AFB, NM 87117-5776

10. SPONSOR/MONITOR'S ACRONYM(S) AFRL/RDSS

11. SPONSOR/MONITOR'S REPORT NUMBER(S) AFRL-RD-PS-TR-2014-0045

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited

13. SUPPLEMENTARY NOTES
377ABW-2014-1026 . Government Purpose Rights.

14. ABSTRACT
A light beam carry Orbital Angular Momentum (OAM) has typical wave front and singularity at the optical axis. The corresponding azimuthal phase is weighted by an integer number which fundamentally represents the OAM component along the propagation. These characteristics have great advantages to be employed in signal transmission technique. Extreme large number of the Hilbert spaces would be available; however, OAM states are sensitive for the atmospheric turbulence even in the weak regime 1,2. Paterson describes the generated scattering by the presence of the turbulence. The decoherence effect that associates with Languerre-Gauss beam is considered and the probability of obtaining different OAM measurements is also calculated. Later on, Taylor and Boyd considered optical vortex beams with constant amplitude. Theydetermined the probability to detect a photon with no change in its OAM states among traveling inside the atmosphere. Both studies assume only a phase distortion causes by the atmospheric turbulence with no change in the wave amplitude.

15. SUBJECT TERMS
Orbital Angular Momentum, atmospheric turbulence, Laguerre-Gauss, Photon Orbital Angular Momentum

16a. REPORT	16b. ABSTRACT	16c. THIS PAGE	17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
Unclassified	Unclassified	Unclassified	SAR	16	David Spendel

16. SECURITY CLASSIFICATION OF:

19b. TELEPHONE NUMBER *(Include area code)*

Standard Form 298 (Rev. 8/98)
Prescribed by ANSI Std. Z39.18

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1.0 SUMMARY

The work reported here includes three investigations related to the study and measurement of orbital angular momentum (OAM): (I) a new characterization of the influence of atmospheric turbulence on light OAM, (II) a study of interferometric OAM measurement error, and (III) an initial look at the transfer of orbital angular momentum (OAM) from photons to electrons.

2.0 INTRODUCTION

Photons can carry both spin and orbital angular momentum (OAM). [1] Spin angular momentum is associated with the polarization of the optical field whereas OAM involves a singularity associated with a dark spot in the intensity distribution and an azimuthal dependence of the wave phase front. The azimuthal dependence is typically a helical wavefront and is mathematically given by a complex exponential term with multiple OAM states or topological charges.[2-3]. This singularity structure is often called an optical vortex. The polarization of a photon provides a two dimensional Hilbert space. But in principle, OAM is associated with the spatial distribution of the wave function and has an infinite number of eigenstates.[4-8] Thus, OAM can be exploited in optical communication applications to increase the information carrying capacity of a beam. Optical beams that involve OAM include Laguerre-Gaussian beam modes, Bessel beams, Hermite-Bessel beams, Airy beams and helical Mathieu beams.

Encoding information for optical communications is a direct application of OAM. In this case, the OAM of an optical beam or wave is artificially manipulated. However, OAM characteristics also arise naturally in other situations such as the propagation of an optical field through atmospheric turbulence.[9] Interaction with the refractive index variations of turbulent air can result in the creation of branch points pairs (two coupled, counter rotating helical phase structures) in the propagated field. These branch points are indicative of optical OAM. Recently, the type of field that leads to the formation of a pair of branch points has been identified.[10] Furthermore, the abundance and spacing of branch point pairs in the received optical field can be related to the turbulence strength and extent.[11] This relationship can be studied to gain a better understanding of turbulence and improved mitigation techniques, or it could even be applied as a diagnostic measurement approach.

Although the singularity points (zero intensity) associated with OAM are not directly measurable due to the finite size of a measurement pixel, the helical phase around the point can be observed with interferometric or wavefront sensing approaches. Measurement methods that have been proposed include: (a) conversion of the helical wavefront to a planar wavefront using a holographic element and then focusing the result through a pinhole or to a fiber optic,[12] (b) circular dichroism angle resolved photoemission;[13] (c) interfering the input wave with a inverted or rotated copy of itself. This approach has been developed into a concept of cascaded of Mach-Zehnder interferometers for measuring the OAM of single photons;[14] (d) in simulation a more complex computer-generated hologram has demonstrated that can detect several different states but with an efficiency that cannot exceed the reciprocal of the number of states.[15] A typical example of an interferometric approach related to methods (a) and (c) and (d) is the concept of interfering a beam with a complex phase front given by $\exp(im\varphi)$ with its mirror image. This produces an interferogram with $2m$ radial spokes. A computer-generated hologram with a fork-dislocation introduces a helical phase front in the diffracted beam. This

type of hologram can be used to create a beam with OAM or, when operated in reverse, the hologram can flatten an input helical phase fronts. These techniques allow photons to be tested only for particular OAM states but they involve tabletop-sized interferometric setups and elements such as holograms, dove prisms and beam splitters. The efficiency of the holographic approaches also tends to be low.

3.0 METHODS, ASSUMPTIONS, AND PROCEDURES

The first stage of this award was to review the fundamentals of photonic angular momentum including the classical view in the paraxial regime as well the quantum principles of light angular momentum. The two components of the optical angular momentum are the intrinsic part (spin component) and the extrinsic part (orbital component). We characterized the features of the most common beam types that exhibit orbital angular momentum (OAM) such as the Laguerre-Gauss (LG) modes. This study also involved a brief review of the manipulation of OAM such as a hologram, spiral phase plate, cylindrical lens, spatial light modulator and q-plates. Pervious work in the literature also includes experimental results of spin-orbit coupling at single-photon entanglement and quantum transfer as well as their combinations. Some studies exist on hybrid entanglement.

3.1 *Characterize the influence of atmospheric turbulence on light OAM.*

We analyze several results of the sensitivity of OAM state due to atmosphere turbulence. The de-coherence effect on optical OAM and the probability of change of its state are characterized. Two example beam modes are considered: LG and vortex beam with constant amplitude. A general formulation is developed that describes the evolution of the rotational field correlation (RFC) as the beam passes through turbulence. We also determine the probability of scrambling of OAM states. The general influence of atmospheric turbulence on light OAM is briefly described. Our procedure can be used to determine the probability of generating specific OAM states due only to the turbulence.

A general model have been presented to describe the propagation of a light beam carrying a certain OAM state through atmospheric turbulence as illustrated in Fig. 1. Paraxial light beam is employed to describe the turbulence influence on the RCF. The extended Huygens-Fresnel principle is applied to a light wave $\psi(\vec{r})$ traveling from a transmitter to a receiver located at the far-field distance z. We assume a circular symmetric source, $|\vec{r}_1| = |\vec{r}_2|$, where $\vec{r}$ is the radial position vector at the transmitter plane, $\Delta\theta$ is the azimuthal angle at the same plane as shown in Fig 1.

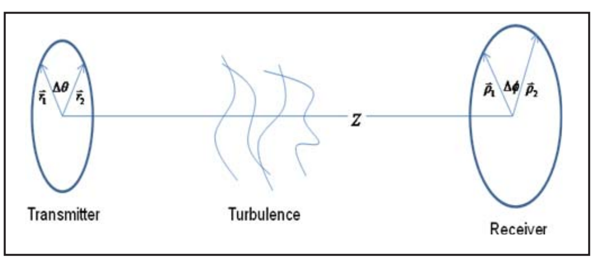

Figure 1. Schematic diagram for OAM propagation through turbulence.

If a homogenous Kolmogorov spectrum is also considered for the turbulence, the received RCF, $C_\psi(\vec{\rho},z)$, is given by

$$C_\psi(\vec{\rho},z) = \int d^2 r\, C_\psi(\vec{r},0) G(\vec{r},\vec{\rho},k,z)\exp\left(-D(\vec{r},\vec{\rho})\right) \quad (1)$$

where $C_\psi(\vec{r},0)$ is the RCF at the transmitter, $G(\vec{r},\vec{\rho},k,z)$ is an appropriate Green's function for free propagation, and $D(\vec{r},\vec{\rho})$, is the wave structure function.

3.2 Interferometric OAM measurement error.

Classical methods for detecting OAM in a coherent beam of light primarily involve the transformation of the spiral phase front of the beam to a plane wave and then examining the "tilt" of the plane wave in the far-field. This concept applies to methods that spatially unwrap the spiral phase or convert the spiral phase through a holographic plate. In these approaches there is sensitivity to the alignment of the collection element (spiral mirror or hologram) to the singularity in the vortex beam (location of zero intensity). We constructed a computer model that simulates the detection of OAM with this unwrapping concept but in the presence of sensor misalignment.

The zero radial mode of the LG beam with azimuthal mode index l is considered. The field amplitude can be represented in polar coordinates as:

$$U_0^l(r,\phi) = A\left(\frac{r}{W(z)}\right)^l \exp\left(-\frac{r^2}{W^2(z)}\right)\exp\left(-jl\phi\right) \quad (2)$$

Where A is an arbitrary constant, $W(z)$ is the radius parameter at a distance z from the beam waist, r is the radial distance from the center axis of the beam, ϕ is the azimuth variable, and z is the axial distance from the beam's narrowest point (beam waist).

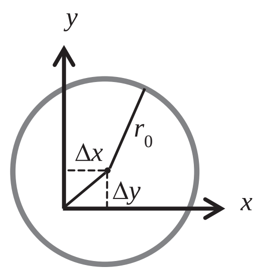

Figure 2. Measurement annulus of radius r_0 is off-centered with respect to beam center in the x direction by Δx and in the y direction by Δy.

If the origin of the measurement annulus is offset with respect to the center of the light beam, as shown in Figure.2, then the resulting Fourier peaks can shift or even bifurcate. Using the polar coordinates $x = r_0 cos\phi$ and $y = r_0 sin\phi$ for a fixed ring of radius r_0, but with a center offset of $(\Delta x, \Delta y)$, the LG beam field on the ring can be written [16]

$$U_0^l(\phi) = A\left(\frac{\sqrt{(\Delta x + r_0\, cos\phi)^2 + (\Delta y + r_0\, sin\phi)^2}}{w}\right)^l \times \exp\left(-\frac{(\Delta x + r_0\, cos\phi)^2 + (\Delta y + r_0\, sin\phi)^2}{w^2}\right)\exp\left[-jl\, tan^{-1}\left(\frac{r_0\, sin\phi + \Delta y}{r_0\, cos\phi + \Delta x}\right)\right] \quad (3)$$

Finding an analytical Fourier transform for Eq. (3) is challenging. Therefore, we computed the discrete Fourier transform of Eq. (3) numerically for a variety of conditions.

3.3 Transfer of orbital angular momentum (OAM) from photons to electrons.

The purpose of this study is to investigate the exchange of angular momentum through interaction of light with an electron. This type of interaction may lead to new ways to detect and measure photonic orbital angular momentum. For our work, the Compton scattering by an angle θ as illustrated in Fig. 3, is applied to a vortex light beam. A photon in a vortex beam possesses spin angular momentum associated with the polarization and orbital angular momentum. We consider the inelastic collision of a photon possessing angular momentum with a free electron. The conservation of angular momentum as well as total energy is applied to the photon-electron system to generalize the Compton scattering model. We describe the momentum exchange and characterize the Compton effect beyond the well-known photon wavelength shift, $\lambda - \lambda'$, to include other parameters such as the radius of gyration.

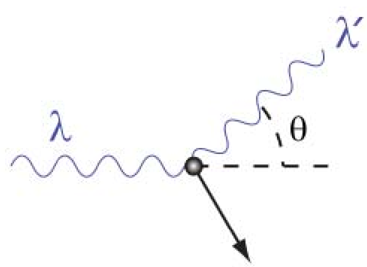

Figure 3. Schematic of Compton scattering with associated parameters.

Our analysis suggests that upon an exchange of angular momentum with an electron, it is possible for the scattered photon to have no wavelength shift.

4.0 RESULTS AND DISCUSSION

The following subsections describe the primary results achieved for the three topics introduced in Section 3. More details can be found in the articles that were published on these works [17-20].

4.1 Characterize the influence of atmospheric turbulence on light OAM.

The exact analytical formula for evaluating the far-field propagation of the RFC through atmospheric turbulence is given by a Hankel transform of the radial part of the spatial structure of the light wave, $|R(r)|^2$, multiplied by a rotational coherence function. Our results include a calculation of the probability of the change of an OAM state due to turbulence strength. The turbulence strength is represented by D/r_0 where D is the receiving aperture diameter and r_0 is the turbulence coherence diameter. Figure 2 illustrates the numerical evaluation of such probability of a vortex beam with constant amplitude [17].

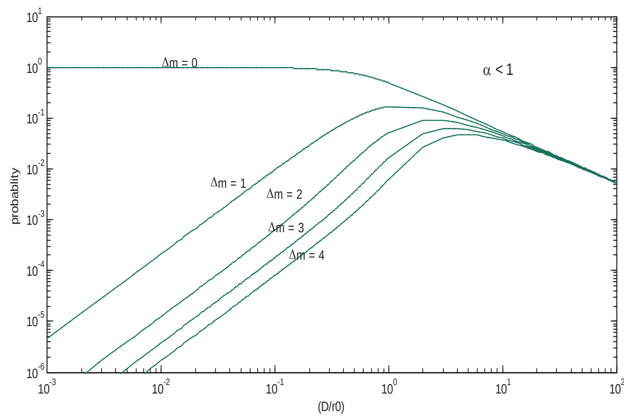

Figure 4. Probability of change in the OAM state of a vortex beam of uniform amplitude vs. turbulence strength (*D*/*r*0).

Our procedure to evaluate the RFC is more general than previous works. It describes RFC`s propagation from the source to the receiver and includes the effects of diffraction. It is shown the RCF is independent of the OAM states at the transmitter. In other words; the probability of a change in OAM state, is independent of the initial OAM mode. Indeed, it is also shown that the highest probability of creating a first order change of OAM is when the Rytov variance ≈ 0.1 which is consistent with the results of a series of experimental measurements [9, 10] where OAM was consistently detected

4.2 Interferometric OAM measurement error.

The discrete Fourier transform is applied to the vector of field points and the magnitude of the result is computed using Eq. (3). A search is done for the highest peak value and the frequency component corresponding to the location of the peak is saved. Figure 5 was generated by selecting $r_0/w = 0.5$ and then repeating the above procedure for normalized offsets ranging from $\Delta x/w = \Delta y/w = -0.5$ to 0.5.

Figure 5 shows the peak position ($-1/2\pi$) predicts the azimuthal mode index $l = 1$, but fails after the offset exceeds a value of about ±0.35. The offset value at which the prediction starts to fail depends on the mode index $l$and the ratio r_0/w. More cases and discussion can be found in reference [16].

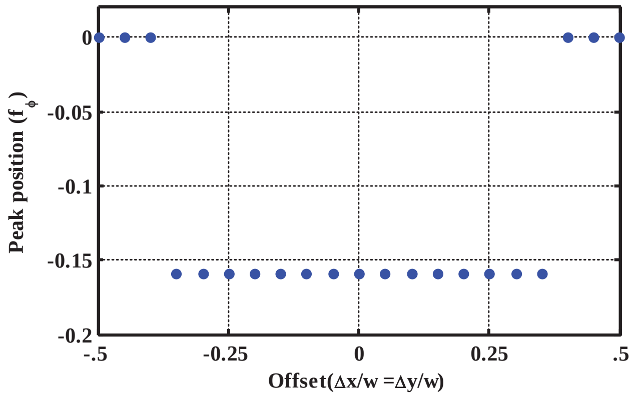

Figure 5. The peak position corresponding to the incident beam offset in both *x* and *y* directions for *l* = 1 and the ratio R_0/w = 0.5.

4.3 Transfer of orbital angular momentum (OAM) from photons to electrons.

A general formula for Compton scattering of a vortex beam by a free electron is derived. The variation of photonic OAM states, $L \rightarrow L'$, is given by[17]:

$$(\eta^2 - 1)L^2 + (\eta'^2 - 1)L'^2 + 2\frac{mc}{\alpha_e}(\eta L - \eta' L') = 2LL`(\eta\eta' - \cos\theta) \tag{4}$$

where η is the normalized dimensionless ratio of linear to angular momentum, m is electron`s mass, $c$ is the light`s speed, and α_e is the ratio of linear momentum to OAM. More specific details are provided in [17] to describe unconventional Compton scattering, which may cause the light angular momentum to change. A change in photonic OAM produces a change in wave number and linear momentum of the light.

5.0 CONCLUSIONS

A general formula was provided that describes the propagation of the RFC through atmospheric turbulence. The formula can predict the change of an OAM state for a single photon as well as the chance of creating certain OAM states in the process of propagating through turbulence.

The sensitivity to the alignment of the annulus of a vortex beam relative to the sensor axis was examined with the general goal of investigating the detection of OAM in the presence of sensor misalignment. Our results indicate that the vortex beam OAM mode can be detected if the alignment is within a reasonable faction of the overall beam size.

The Compton scattering momentum formula was generalized to include the conservation of light angular momentum. A generalized Compton scattering model of a vortex beam describes the momentum exchange beyond the well-known photon wave number shift. Our analysis suggests that upon an exchange of angular momentum with an electron, it is possible for the scattered photon to have no wavelength shift.

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APPENDIX- Presentations and Publications Resulting from this Research.

A. M. Nairat and D. Voelz, "Propagation of the Optical Rotational Correlation Field and Orbital Angular Momentum through Turbulence," in Imaging and Applied Optics, OSA Technical Digest (online) (Optical Society of America, 2013), paper PTu3F.5.

B. M. Nairat and D. Voelz, "Propagation of rotational field correlation through atmospheric turbulence," Opt. Lett. 39, 1838-1840 (2014).

C. M. Nairat and D. Voelz, “Compton Scattering with a Vortex Light Beam,” Conference presentation for the American Physical Society Meeting, Denver, CO, Mar. 3-7, 2014.

D. N. Jayasundara, D. Voelz, and M. Nairat, "Alignment Considerations for Optical OAM Detection," in Imaging and Applied Optics 2014, OSA Technical Digest (online) (Optical Society of America, 2014), paper PW1E.4.

E. M. Nairat and D. Voelz, "Using quantum operators to describe orbital angular momentum of Hermite Gaussian modes," in Imaging and Applied Optics 2014, OSA Technical Digest (online) (Optical Society of America, 2014), paper PW1E.5.

LIST OF ABBREVIATIONS

LG	Laguerre Gauss
OAM	Orbital Angular Momentum
RFC	Rotational Field Correlation

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